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UDC 371.3:511.13
Teaching Innovations, 2015, Volume 28, Issue 3, pp. 15–35 371.315.1/.4

Paper received: June 15th 2015
Paper accepted: October 9th 2015

Original Paper

DOI:10.5937/inovacije1503015B

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Abstract: *In primary school, learning fractions is a central mathematical objective. However, the mastery of basic procedures involving fractions presents a difficulty for many students. The aim of the current intervention is to introduce structured cooperative learning as means to improve students' learning, particularly for average achievers. Previous research has underscored that heterogeneous groups might be deleterious for average achievers because they are excluded by the teacher learner relationships that is likely to take place between low and high achievers students. This intervention proposes structuring interactions in order to boost the learning of average achievers in heterogeneous groups. We hypothesize that highly structured cooperative learning should improve average achievers' understanding of the content-targeted in group work as well as progress in terms of fractions learning, when compared to low-structured cooperative learning.*

In this intervention, 108 fifth graders worked cooperatively in heterogeneous triads (a low, average, and high achiever). The triads had to express the length of one segment using three rulers with different sub-units

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and respecting three mathematical skills regarding fractions. Triads were randomly assigned to a low-structured or high-structured cooperative learning condition. In the low-structured condition, no specific structure was provided. (i.e., they organized their cooperative work as they wished). In the high-structured condition, each student became an expert for one part before working in the triad and endorsed different responsibilities.

The results indicated that highly structured cooperative learning favors the understanding of the targeted task, especially for average-ability students. Moreover, students at all levels progressed from the baseline test to the post-test. Indeed, low and high achievers had the same progression in both conditions, whereas average achievers progressed more in the highly structured condition. Results are discussed in terms of new teaching methods that could efficiently increase average achievers' performances.

***Key words**: Cooperative learning, structure, fraction learning, average achievers, mathematics.*

Introduction

In most countries, mathematics is considered one of the most important topics to learn in primary school (Joët, Usher, & Bressoux, 2011; OECD, 2009; Yusof & Malone, 2003). Fractions represent a fundamental cornerstone for the understanding of advanced mathematical concepts, such as algebra, geometry, and statistics (Bailey, Hoard, Nugent, & Geary, 2012). Learning fractions requires deep procedural and conceptual knowledge (Rittle-Johnson & Alibali, 1999) that enables students to thoroughly understand and distinguish between the properties of whole numbers and rational numbers (Ni & Zhou, 2005). Previous work (Siegler et al., 2012) has demonstrated that knowledge of fractions in elementary school predicts competence in general mathematics and algebra in high school.

Despite their undoubted importance in mathematics, fractions remain one of the toughest concepts. The mastery of basic procedures about fractions still represents a difficulty for many students (Carette, Content, Rey, Coché, & Gabriel, 2009; Lin, Wenli, Lin, Su, & Xie, 2014). The National Council of Teachers in Mathematics (Martin & Strutchens, 2007) reported that only 50% of American 8th graders are able to put a series of fractions in the correct order. Furthermore, it seems that the obstacles and deficiencies in fraction knowledge are persistent (Mazzocco & Devlin, 2008). In the present research, we focus in particular on fractions learning among 5th graders of different abilities (low versus average versus high).

To address these difficulties, it is particularly important to design teaching methods and intervention programs that could enhance students' understanding of fractions and tackle low school achievement (e.g., Gabriel, Coché, Szucs, Carette, & Rey, 2012). The aim of the present intervention is to test cooperative learning as a way to improve the understanding and learning of fractions. Moreover, we intended to compare two different forms of cooperative learning—namely, low versus high structured—with respect to the level of students working in heterogeneous teams. Regarding this issue, the prevailing recommendation for the implementation of cooperative learning involves wide range heterogeneous grouping (with high, average, and low achievers in the same group; see Abrami et al., 1995; Sharan, 1999). Nevertheless, research underscores that working in wide-range heterogeneous groups might be problematic for the average students. Indeed, average achievers tend to be less active in this particular group composition (Webb, 1991). It is thus essential to consider a way of maximizing the benefits of cooperative learning in heterogeneous groups for all students. We argue that highly structured cooperative learning might stimulate all students' involvement in wide range grouping and be especially positive for average achievers (Saleh, Lazonder, & de Jong, 2007).

Cooperative Learning

Basic Principles for Cooperative Learning

Cooperative learning is a teaching method in which students work cooperatively in small groups in order to enhance their own and their peers' learning (Abrami, Poulsen & Champer, 2004). A substantial body of research has pointed out the benefits of this practice on students' learning, productivity, social relationships, motivation, and self-esteem (Gillies, in press; Johnson & Johnson, 2009; Johnson, Johnson, Roseth, & Shin, 2014; Slavin, 2014).

Cooperative learning work—compared to unstructured group work—should be organized to ensure its effectiveness (Gillies, 2003, 2007; Johnson, Johnson, & Holubec, 2008). Two principles are essential in all cooperative methods (see Sharan, 1999): positive social interdependence and individual responsibility. Positive social interdependence implies that students' outcomes are affected by their own and others' actions (Johnson & Johnson, 2005). This interdependence can be structured in various ways within a group (Johnson & Johnson, 1989; Johnson, Johnson, & Holubec, 1998). It requires students to work towards a common goal, and they perceive that they can achieve this goal only if all the members of their group attain their individual goals. This positive goal interdependence can be defined in terms of either a joint product or the mastery/learning of all members. Positive interdependence can be reinforced by other dimensions (Johnson, Johnson, & Holubec, 1993), such as sharing complementary resources, being responsible for a delimited part of the task, or endorsing a specific responsibility. Individual responsibility involves each member contributing and being held accountable for his/her own learning and that of others (Johnson et al., 2008; Kagan & Kagan, 2000). Assigning specific roles to team members, identifying each other's contributions, and assessing individual learning are some of the ways that individual responsibility can be increased (Bennett, Rolheiser, & Stevahn, 1991).

Finally, both positive interdependence and individual responsibility favor the development of constructive interactions (Davidson, 1994; Johnson & Johnson, 2009). Students are required to exchange ideas as well as share knowledge and learning strategies (Leikin & Zazlavsky, 1999). They should encourage and teach each other (Battistich, Solomon & Delucchi, 1993), discuss their agreements, and elaborate on their conflicts (Buchs, Butera, Mugny, & Darnon, 2004). These interactive processes favor understanding and learning (Johnson et al., 1998; O'Donnell & King, 1999). Working cooperatively with other peers, students have to verbalize and make visible their knowledge and their reasoning (Mercer, Wegerif, & Dawes , 1999). Based on this, peers are likely to detect what is not understood by their partners and to give understandable explanations (Gillies & Ashman, 1998) that are positively related to gain in sciences understanding (Howe et al., 2007) and performance in mathematics (Webb, 1991). ; Argumentation permits students to reach a shared understanding and favors emergent learning during argumentative talk as well as learning following argumentative interactions (Schwartz, 2009).

Benefits of Cooperative Learning for Mathematics

Over the last few decades, cooperative practices have gained significant grounds in mathematics achievement. Several studies have indicated the superiority of cooperative learning in mathematics over traditional practices—namely, individual work and competition (e.g., Zakaria, Chin, & Daud, 2010). Cooperative learning is linked to positive attitudes toward mathematics and achievement (Zakaria et al., 2010; Tarim & Akdeniz, 2008; Walmsley & Muniz, 2003), problem-solving strategies (Duren & Cherrington, 1992), and fractions learning (Lin, Chen, Lin, Su, & Xie, 2014).

Cooperative learning is supposed to be particularly beneficial for learning mathematics because it supports thinking rather than producing answers, develops multiple representations, accommodates different learning styles, and reduces students' anxiety (Bassarear & Davidson, 1992). Leikin and Zaslavsky (1997) pointed out that cooperative settings facilitated students' activeness and mathematical communications (e.g., asking questions, giving explanations, and requesting help). Giving related-content explanations and observing other group members interacting are positively related to mathematic achievement (see Webb, 1991, for a review). Furthermore, receiving elaborated help contributes to the learning of mathematics on the condition that the received explanations are elaborated on and used subsequently in a constructive problem activity (e.g., problem-solving; Webb, Troper, & Fall, 1995).

Importance of Structuring Cooperation in Heterogeneous Groups

The implementation of cooperative learning has been inextricably linked to heterogeneous group composition by a significant number of researchers and manuals (e.g., Davidson, 1990; Abrami et al., 1995; Sharan, 1999). Nevertheless, scholars do not agree on the benefits of heterogeneous grouping (e.g., Lou et al., 1996). Taking into account the interactions that occur in groups can help better understand the effect of group composition (Fuchs, Fuchs, Hamlett, & Karns, 1998). Indeed, empirical evidence suggests that grouping influences the degree to which different achievers (low, average, high) respond and participate within a group (Saleh, Lazonder, & De Jong, 2005; Webb, 1991). For instance, low-ability students perform well in heterogeneous groups in which they have the possibility of interacting with more competent individuals, asking questions, receiving explanations, and filling in the gaps in their knowledge (Lou et al., 1996; Hooper & Hannafin, 1991). As far as students with high ability are concerned, they can benefit from both heterogeneous and homogenous groups (Lou et al., 1996; Saleh et al., 2005). Finally, average-ability students seem to be the least favored in wide range heterogeneous groups. They tend to stand back, participate less, and are excluded from the peer–tutee relationship that often takes place between high- and low-ability students (Saleh et al., 2005; Webb, 1991).

Interestingly, however, research has shown that average achievers working with only low achievers (low and average students) or with high achievers (average and high students) are more active and perform better compared to when they work in wide-range heterogeneous grouping with low, average, and high students (Hooper, 1992; Webb, 1991). Moreover, Saleh and colleagues (2007) indicated that additional support is needed to strengthen verbal interactions and the learning of average-ability students in wide-range heterogeneous groups. In their study, they provided ground rules for helping to facilitate elaborate explanations in the groups. More importantly, they introduced rules to prevent the same students from initiating all explanations. The objective was to force average achievers to take a more active role in explanations in heterogeneous groups (1 high achiever, 2 average achievers, and 1 low achiever). This structure favored learning for all students and enhanced the motivation as well as the participation of average students.

Thus, taken together, these results suggest that wide-range heterogeneity might be detrimental for students in an intermediate position while activating the peer–tutee interactions between low and high achievers. However, they point to the fact that the intermediate position is not an obstacle *per se*. Indeed, when these students have the opportunity to exchange ideas with their peers (for example, when they only interact with a low- or high-ability partner or when cooperation is highly structured), they can benefit from cooperation. Thus, a crucial question emerges: How can cooperation be organized to make sure each student, including average students, can actively participate in the discussion and benefit from cooperation?

Many researchers have underscored the need to structure carefully cooperative learning (Gillies, 2004, 2008; Webb, 2009) and help students cooperate (Blatchford, Kutnick, Baines, & Galton, 2003; Tolmie et al., 2010) in order to promote constructive interactions. Notably, it is important to establish positive norms for cooperative work and constructive behaviors (Webb, Farivar, & Mastergeorge, 2002) and create conditions for simultaneous interactions that foster contributions from all team members (Kagan & Kagan, 2000). Proposing scripts for interactions (O'Donnell & Dansereau, 1995; Schellens, Van Keer, De Wever, & Valcke, 2007), explicit trainings regarding interpersonal and collaborative skills (Gillies, 2003), or rules for stimulating participation and helping (Saleh et al., 2007) can be effective ways to stimulate interaction and learning. Gillies and Ashman (1995) found that the effect of ability composition is minimal in structured cooperative groups. The present study aims to test whether highly structured cooperative learning can boost average achievers' learning in cooperative groups.

Overview of the Present Research

Considering that fractions remain a major difficulty for pupils in primary school, the first purpose of our intervention was to introduce cooperative learning as a way to favor learning in fractions. We argue that a general cooperative framework can offer a good opportunity for students to increase their mastery of fraction procedures and permit some progress in terms of fraction learning. Thus, in all groups, primary pupils were led to work in triads on a fraction exercise. The instructions involved three cooperative principles: positive interdependence, individual responsibility, and constructive interactions. Indeed, pupils were asked to help each other to master three mathematical skills in order to reach a common answer and to ensure that all the team members understand. They were informed that they would answer an individual learning test after the group work. In the low-structure cooperative learning condition, no additional instruction was provided.

To address the issue of wide heterogeneity in groups (with low, average, and high achievers), another condition was designed. Indeed, starting from the premise that average achievers might be less active in such groups and that taking an active role in giving explanations is a crucial element in mathematics, the highly structured cooperative learning condition intended to ensure that all students in the teams would be engaged in mathematical discussions and group decisions. To that end, positive interdependence was reinforced through resource distribution, complementary expertise, and alternated responsibilities during the exercise. We hypothesized that highly structured cooperative learning should improve all students' understanding and learning of fractions and should be particularly beneficial for average achievers, compared to low-structured cooperative learning.

Method

Participants

One hundred eight 5^{th} graders from seven primary schools participated in this intervention study. Pupils were divided into 36 working groups of three. Preliminary analyses revealed one influential group that could be considered as deviant and, thus, was dropped from the analyses (Cooks' D > .14; Snijders & Berkhof, 2008).[2] The final sample comprised $N = 105$ pupils, embedded in $k = 35$ triads and $l = 9$ classes (49 girls and 56 boys, $M_{age} = 10.66$, $SD = 0.58$).

Procedure

Parental consent was requested, and anonymity was guaranteed. Teachers were present except during group work. The intervention took place over two sessions in pupils' classrooms (see Table 1).

2 It should be noted that the hypothesized results remained roughly the same when keeping this influential group—namely, χ^2 (2, $N = 104$) = 7.04, $p = .029$ for understanding, and χ^2 (2, $N = 104$) = 7.04, $p = .086$ for learning.

The didactic objective proposed for the group work was derived from a standardized national evaluation on fractions (see, French Ministry of National Education, 2008). The mathematical task involved three skills: 1) understanding fraction reasoning (the addition of a whole number + fraction, the addition of fractions, the fractional writing); 2) figuring out the equivalence of the writings for different reasonings $\left(1+\frac{1}{3}\right)$, $\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)$ and $\left(1+\frac{4}{3}\right)$; and 3) being able to use adequate vocabulary. In order to work on fraction notions, we proposed typical exercises used in the national curriculum.

Table 1. *Summary of the procedure*

Session 1

Baseline test (*9 fraction exercises).*

Lessons and exercises with three mathematical targeted skills:

- *explaining the three reasoning*
- *verifying the equivalence of the writing*
- *communicating with appropriate vocabulary*

Session 2

General cooperative learning instructions. The three mathematical targeted skills are reminded. A visual support introduced the three social responsibilities

Pupils worked in heterogeneous triads, randomly assigned to one or the other of the experimental conditions.

Low-structure condition	**High-structure condition**
15 min.: Each pupil of the triad worked individually with the three rulers $\left(\frac{1}{4}\right), \left(\frac{1}{8}\right), \left(\frac{1}{16}\right)$.	***10 min.*****:** Each pupil of the triad worked individually with one of the three rulers. ***5 min.***: pupils were grouped with others who get the same ruler in order to get a common solution (expert groups).
10 min.: Pupils worked in triad. They organized the group work as they wished in the respect of the three mathematical skills and the three social responsibilities.	***10 min.***: Pupils worked in triad. Each pupil of triad was responsible of one mathematical skill and one social responsibility at time; responsibilities rotated so that all pupils endorsed all skills at one time.

Individual understanding (pupils *individually performed a fraction exercise, similar to those carried out in their triads but with a new ruler).*

Standardized post-test measure (*9 fractions exercises).*

First session. In the first session, pupils individually performed the baseline test covering the whole notion of fractions. After this test, the experimenter made a lesson on fractions and gave two specific fraction exercises for the pupils to solve collectively. Three relevant mathematical skills identified by the National Mathematics Program (Ministère de l'Education Nationale, 2008) were targeted in this exercise: explaining the reasoning, verifying the equivalence of the writing, and communicating with appropriate vocabulary. The lesson allowed the experimenter to provide the exact same amount of information about fractions to all pupils. This included oral explanations and visual supports (displayed on the board during the entire intervention).

Second session. One week later, pupils worked in triads on fraction exercises. In both conditions (low- and high-structure conditions), the experimenter started by reminding the students of the three mathematical skills (explaining reasoning, checking the equivalence of writing, using adequate vocabulary) through visual supports, which remained available throughout the session in the classroom. The experimenter then introduced general cooperative learning instructions for all pupils: She asked pupils to work in triads with a focus on learning and mastery. Pupils were instructed to work cooperatively, taking care of their own learning and their partners' learning. Three social responsibilities were also enhanced: checking that everyone understood; verifying that everybody agreed on the common answer, and reporting the common answer. Pupils reported their consensual answer on the group sheet (positive goal and resource interdependence). They were asked to encourage each other and explain their reasoning (constructive interactions). They were also informed they would complete an individual learning test after the group work (individual responsibility). These cooperative instructions were provided in both conditions.

Pupils were assigned to the different triads according to their performance on the standardized baseline test. Specifically, within each class, each pupil was placed in a heterogeneous triad with one low, one average, and one high achiever. The task consisted of one exercise on fractions adapted from two pedagogical books for 5th grade (Briand, Vergnes, Ngono, & Peltier, 2009; Charnay, Douaire, Valentin, & Guillaume, 2005). These exercises had to be solved in triads and consisted of presenting a segment to pupils. They were asked to use a standard measure in order to express the length of this segment in terms of fractions of a standard measurement.

The standard measure was graduated with different sub-units, respectively representing $\left(\frac{1}{4}\right)$, $\left(\frac{1}{8}\right)$ and $\left(\frac{1}{16}\right)$, which we named "the three rulers." Pupils had to write the length of the segment using as many writings as possible while using adequate vocabulary. They also had to check that all writings were equivalent. They were required to use all rulers to measure the segment. During this phase, the degree of structure varied depending on the conditions: low- versus high-structured cooperation (see Independent Variables).

After the exercise in triads, individuals' understanding was evaluated (see Dependent Variables), and then pupils resolved an individual posttest covering the whole notion of fractions (see Dependent Variables).

Independent Variables

Initial level of achievement. The baseline test consisted of nine fraction exercises extracted from French standardized national assessments and from a previous study (Carette et al., 2009). This baseline test lasted 20 minutes. Theoretically, scores can range from 0 to 20. Depending on their score at the baseline test, pupils were considered low achievers ($M_{pre\text{-}test}$ = 5.23, SD = 2.65), average achievers ($M_{pre\text{-}test}$ = 10.65, SD = 3.04), or high achievers ($M_{pre\text{-}test}$ = 14.94, SD = 2.98).

Structure of cooperation. In each class, half of the pupils were randomly assigned to a low-structure cooperative learning condition, whereas the other half was assigned to a high-structure cooperative learning condition. In the *low-structure condition* (n = 51, k = 17, l = 9), material was distributed to all pupils (i.e., each pupil had the three different rulers $\left(\frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right)$). Pupils had to apply the three mathematical skills (explaining reasoning, checking the equivalence of writing, and using adequate vocabulary). They individually worked on the exercise for 15 minutes with the three rulers. After this work, they had to discuss their answers in their triads, using all skills and rulers; they had to make sure that everybody understood and then report their consensual answers. They organized their group work however they wished (10 minutes).

In the *high-structure condition* (n = 54, k =18, l = 9), materials were divided among the pupils in each triad (i.e., one ruler per person), reinforcing the positive resource interdependence. Pupils worked alone with one ruler for 10 minutes. They were then grouped with other pupils with the same ruler (i.e., in "expert groups") for 5 minutes; they interacted with all the pupils from their session who had received the same ruler as they did. Their goal was to find a common solution. After this expert group work, pupils returned to their original triads and had to explain their acquired skills to their peers. We introduced specific responsibilities based on the targeted mathematical skills and the targeted social responsibilities introduced in the general cooperative framework and we proposed that pupils alternate these responsibilities during the exercise. Thus, when working with the first ruler, one of the pupils was responsible for explaining his/her reasoning (mathematical skills) and for ensuring that everybody understood (social responsibility); the other pupil was responsible for checking writing equivalence (mathematical skills) and that everybody agreed (social responsibility); and the third pupil was responsible for checking that all partners used adequate vocabulary (mathematical skills) and for reporting the common answer on the group sheet (social responsibility). For the second and third rulers, responsibilities were rotated so that each pupil was required to endorse all responsibilities at one time. In order to help pupils organize their responsibilities, they could rely on a summary card (see Appendixes A, B, and C). Each card contained the visual support for mathematical skills (those proposed in the collective lesson and displayed on the board in all conditions) and some words to help pupils with social responsibility. This procedure was proposed to reinforce both individual responsibility and positive interdependence.

Dependent Variables

Individual understanding. After the group work, pupils individually performed a similar fraction exercise as those carried out in their triads, but with a new ruler (adapted from Briand et al., 2009; Charnay et al., 2005). In this application exercise, they were asked to measure the length of a segment with a new ruler graduated in $\left(\frac{1}{5}\right)$. Mean grades could range from 0 to 3 (M = 1.88, SD = 1.29). Zero points were assigned for a non-answer or a false or incomprehensible answer. One point was allocated for correct answers without using fractions, two points for at least one correct answer using fractions, and three points corresponded to several correct answers using fractions.

Individual progress in fractions learning. Individual progress in fraction learning was measured by assessing the evolution from baseline test to post-test. The baseline test and the post-test covered the whole notion of fractions. They consisted of 9 fraction exercises extracted from standardized national assessments and from a previous study (Carette et al., 2009). The two tests were the same except that all mathematical values were changed. They were corrected by the experimenter, who remained blind to the experimental conditions. The same standardized evaluation matrix was used to compute an individual's score, theoretically ranging between 0 and 20 (mean scores for baseline M = 10.31, SD = 4.88; mean scores for post-test M = 13.52, SD = 4.52; observed mean progress M = 3.21, SD = 3.38).

Table 2. *Coefficients estimating and statistical tests of the multilevel models testing the effect of the initial level of achievement and the structure of cooperation on individual level of understanding (first set of analyses) and learning (second set of analyses).*

		First set of analyses: Understanding			Second set of analyses: Learning		
		B	CI	Test	*B*	CI	Test
Level 1	Intercept, $\hat{a}_{000}$	1.85	1.50, 2.20	$Z = 10.33^{**}$	3.19	2.60, 3.77	$Z = 10.70^{**}$
	Initial level of achievement (IAch), $\hat{a}_{100}$	n/a	n/a	$\div^2 = 44.00^{**}$	n/a	n/a	$\div^2 = 21.40^{**}$
	Age (A), $\hat{a}_{200}$	-0.55	-0.88, -0.22	$Z = 3.25^{**}$	-0.97	-2.03, 0.09	$Z = -1.79^{\dagger}$
Level 2	Structure of cooperation (Coop), $\hat{a}_{001}$	0.42	0.07, 0.77	$Z = 2.33^{*}$	0.65	-0.51, 1.82	$Z = 1.10$
Cross-level	Initial level of achievement x structure of cooperation, $\hat{a}_{101}$	n/a	n/a	$\div^2 = 7.96^{*}$	n/a	n/a	$\div^2 = 6.27^{*}$
Residuals	Level-1 variance, $\mathring{a}_{ijk}$	0.79	0.59, 1.05	n/a	8.76	6.26, 12.26	n/a
	Level-2 variance, æ_{0jk}	0.00	n.s.	n/a	0.01	n.s.	n/a
	Level-3 variance, æ_{0k}	0.21	0.06, 0.78	n/a	0.12	n.s.	n/a

Notes: The formula of each model is $Y_{ijk} = \beta_{000} + \beta_{100} * IAch_{ijk} + \beta_{200} * A_{ijk} + \beta_{001} * Coop_k + \beta_{101} * IAch_{ijk} * Coop_k + \zeta_{0jk} + \zeta_{0k} + \varepsilon_{ijk}$; the effects of the initial level of achievement (i.e., a categorical variable with three modalities) were obtained using dummy variables; n/a means "not applicable", and n.s. "non significant"; ** $p < .01$, * $p < .05$, † $p < .1$.

Results

Overview of the Multilevel Regression Analyses

A summary of the results is presented in Table 2. Observations consisted of pupils (i.e., level 1) nested in triads (i.e., level 2) nested in classrooms (i.e., level 3). Given the hierarchical structure of the data, three-level multilevel modeling was employed (Rabe-Hesketh & Skrondal, 2012). Specifically, a first set of multilevel regression analyses was performed using individuals' understanding as the dependent variable; a second one was conducted using individuals' progress in fraction learning as the dependent variable.[3]

In each set of analyses, our dependent variable was regressed on three predictors: (i) the initial level of achievement (i.e., a level 1 categorical variable: low versus average versus high achiever), (ii) the structure of cooperation (i.e., a level 2 dichotomous variable: coded -0.5 for low structure and +0.5 for high structure), and (iii) the cross-level interaction between the two. It is worth noting that, in preliminary analyses, the pupil's age was found to be negatively associated with both individual understanding and learning (cf. Table 1). Hence, grand-mean centered age (i.e., a level 1 continuous variable) was always statistically controlled.

Initial level of achievement, structure of cooperation, and understanding. First of all, a main effect of the initial level of achievement was found, χ^2 (2, N = 104[4]) = 44.00, $p < .001$. Notwithstanding the structure of cooperation, low achievers (M = 1.15, 95% CI [0.72, 1.58][5]) obtained a lower score of individual understanding than average achievers (M = 1.82 [1.38, 2.25]), who themselves obtained a lower one than high achievers (M = 2.58 [2.15, 3.00]).

3 As far as individuals' understanding is concerned, intraclass correlation did not differ from zero to level 2, indicating that the variance of understanding was not due to between-triad differences, and was $\rho = .15$ at level 3, indicating that 15% of the variance of understanding was due to between-class differences. As far as the learning is concerned, intraclass correlation did not differ from zero to level 2 and was $\rho = .09$ at level 3. However, as recommended by Barr, Levy, Scheepers, and Tily (2013), all random intercepts were included in the final model.

4 The sample size is N = 104 (rather than N = 105) because of one missing value on our dependent variable.

5 From here on, the 95% CI is omitted. Hence, all square brackets signal a 95% confidence interval.

Second, a main effect of the structure of cooperation was observed, $B = 0.41$, [0.07, 0.77], $Z = 2.33$, $p = .02$. Compared with the pupils in the low-structure cooperation condition ($M = 1.64$ [1.25, 2.04]), the pupils in the high-structure cooperation condition ($M = 2.06$ [1.67, 2.45]) gave an average of 0.41 (out of three) more correct responses. In other words, higher structure was beneficial for all pupils' understanding, regardless of their initial level of achievement.

Third and more importantly, analyses revealed a cross-level interaction effect between the initial level of achievement and the structure of cooperation, χ^2 (2, $N = 104$) = 7.96, $p = .019$. In other words, depending on the initial level of achievement, the effects of the structure of cooperation were not the same. Average achievers benefitted the most from structured cooperative learning, $B = 1.11$ [0.51, 1.72], $Z = 3.62$, $p < .001$. Average achievers in the high-structure cooperation condition ($M = 2.38$ [1.85, 2.90]) gave an average of 1.11 (out of three) more correct responses than those in the low-structure cooperation condition ($M = 1.26$ [0.73, 1.80]). However, the effect of the structure of cooperation was significant for neither low achievers, $B = 0.01$ [-0.59, 0.60], $Z < 1$, *n.s.*, nor high achievers, $B = 0.13$, [-0.48, 0.73], $Z < 1$, *n.s.* These results indicated that low achievers did not provide more correct answers when cooperation was highly structured ($M = 1.15$ [0.63, 1.68]) than when it was not ($M = 1.16$ [0.63-1.68]). Similarly, for high achievers, no differences were observed between the low-structure cooperation condition ($M = 2.51$ [1.98, 3.04]) and the high-structure one ($M = 2.64$ [2.12, 3.15]. In sum, in line with our hypothesis, and as can be seen in Figure 1, structuring cooperation was particularly beneficial for average achievers' understanding, relative to low and high achievers.

Initial level of achievement, structure of cooperation, and individual progress in fractions learning. As far as the second set of analyses is concerned, we aimed to test our hypothesis using progress in learning as a dependent variable. Hence, we subtracted the performance on the baseline test from that on the post-test; the more positive the computed variable, the higher the improvement. Progress was then regressed on the same predictors as before—namely, (i) the initial level of achievement, (ii) the structure of cooperation, (iii) the cross-level interaction between the two, and (iv) age.

First, the intercept was significantly different from zero, $B = 3.18$, [2.60, 3.76], $Z = 10.70$, $p < .001$. Irrespective of both the condition or the initial level of achievement, it pertained to the fact that pupils progressed an average of 3.18 points (of 20) from the baseline test ($M = 10.27$ [8.89, 11.65]) to the post-test ($M = 13.45$ [12.87, 14.04]).

Second, a main effect of the initial level of achievement was found, χ^2 (2, $N = 104$[6]) = 21.40, $p < .001$. This result indicated that, overall, low achievers made more baseline-to-post-test progress ($B = 5.12$ [4.11, 6.14]) than average achievers ($B = 2.45$ [1.44, 3.47]), who themselves made more progress than high achievers ($B = 1.98$ [0.97, 2.98]). Such a finding might simply reflect that lower achievers have greater room for improvement (due to starting from a lower level). Hence, mechanically, the lower the initial achievement, the stronger the effects of cooperation—be it poorly or highly structured—on improvement.

Finally, an interaction effect between the initial level of achievement and structure of cooperation was once again observed, χ^2 (2, $N = 104$) = 6.27, $p = .044$. Simply put, as a function of the initial level of achievement, the effect of the structure of cooperation was different. As far as average achievers are concerned, the structure of cooperation predicted a progress of 2.64 extra points, $B = 2.64$ [0.66, 4.62], $Z = 2.61$, $p = .009$. Indeed, from the baseline to the post-test, the average achievers in the low-structure condition progressed by $B = 1.14$ [-0.29, 2.56] points, whereas in the high-structure condi-

6 Once again, there was one missing value on our dependent variable; it is not the same participant as before.

tion, they progressed by $B = 3.77$ [2.37, 5.18] points. However, the structure of cooperation did not predict differences in terms of progress for low achievers $B = -0.79$ [-2.77, 1.19], $Z < 1$, *n.s.* It indicated that low achievers progressed the same when cooperation was highly structured ($B = 4.73$ [3.32, 6.14]) or not ($B = 5.52$ [4.09, 6.94]). Furthermore, the structure of cooperation did not predict progress for high achievers, $B = 0.11$, [-1.92, 2.15], $Z < 1$, *n.s.* In other words, once again no differences were observed between the low- ($M = 1.92$ [0.45, 3.39]) and high-structure cooperation conditions ($M = 2.04$ [0.65, 3.42]). In sum, in line with our hypothesis, and as seen in Figure 2, structuring cooperation triggered particular improvements for average (versus low or high) achievers.

Discussion

As mentioned in the introduction, learning fractions remains one of the toughest concepts to learn at school. This paper focused on cooperative learning as a tool to foster learning fractions, especially for average-ability pupils in largely heterogeneous groupings. We argued that, although generally positive for learning, cooperative learning might not be beneficial for intermediate position achievers in heterogeneous groups (low-, average-, and high-ability students). Indeed, these students might suffer from being excluded from the discussion. In the present paper, we argue that structuring cooperation can actively engage each pupil in the group discussion; as such, highly structured cooperative learning might be particularly beneficial for average-ability pupils compared to weakly structured cooperative work. In both conditions, the experimenter introduced cooperative instructions (with positive interdependence, individual responsibility, and constructive interactions). The group work was built around common material (three rulers), mathematical skills (three specific skills), and social responsibilities (three social roles). The main difference between the two conditions was that, in the low-structured condition, pupils organized their work as they wished whereas, in the high-structure condition, materials were divided among pupils and each of them had to endorse specific responsibilities at different moments in the group work. Thus, the present study tested whether high- and low-structure conditions affect individual understanding and individual progress in terms of fractions learning and whether this impact depends on the pupil's initial level.

First, the results indicated that the high-structured condition increased pupils' understanding more than the low-structured condition. This point is important. Indeed, from a pedagogical perspective, this result sustains that structured cooperative learning is more beneficial for mathematical understanding than unstructured cooperative learning, specifically for fractions learning topic. More importantly, statistical analyses demonstrated that more structure mainly increased the understanding for average achievers but did not affect the understanding of low and high achievers. Thus, highly structured cooperative learning seems to be especially efficient for average achievers' understanding.

Regarding individual progress in fractions learning, positive progression is observed in both low- and high-structured conditions for all pupils. Thus, cooperative learning offers some benefits for mathematical (Zakaria et al., 2010) and fractions learning (Lin et al., 2014). This progression is even stronger when pupils' initial level was low. Moreover, as for the understanding variable, the interaction indicated that more structure increased individuals' progress in learning fractions mainly for average achievers. Once again, the degree of structure did not affect individuals' progress for low and high achievers.

Taken together, these findings underscore that more structure (versus less) appears to be more effective for average achievers than for low or high achievers, who might benefit from cooperation whatever its level of structure. The other important point underscored by the present study is that the degree of structure has no effect on either the understanding or the progression of low and high achievers.

These findings suggest that a structure that imposes all students to be socially and cognitively engaged during group work is a crucial component that enables average achievers to benefit from cooperation. This appears to be particularly important in elementary school, where teachers are likely to compose heterogeneous groups (Saleh et al., 2005). Our results indicated that building heterogeneous groups in a class requires special attention on average achievers. Indeed, they underscored the benefits of highly structured cooperative learning for average achievers. Although often excluded from social interactions in classic heterogeneous group work (Saleh et al., 2005; Webb, 1991), cooperative structure might be a solution to balance the interactions among group members. As such, this study proposes an interesting pedagogical cooperative learning method that can be used in classrooms to improve the organization of these interactions in heterogeneous work groups.

Our results suggest that participation in constructive social interactions in cooperative heterogeneous groups may be important and that the structure introduced may favor active involvement from all partners in the group. However, in the present study, pupils' actual participation was not directly measured. Future research could integrate video-taping of the different group work efforts to measure the extent to which average achievers participate in the group discussion more actively in the highly structured cooperative condition than in the low-structured condition.

As previously mentioned, the cooperative learning procedure designed in the present study can be used directly by teachers in their classrooms to develop average achievers' understanding and progress without affecting low and high achievers' performances. It is interesting to note that the present research focused on both individuals' understanding regarding the specific task and generalized progress in learning fractions. However, previous research has documented that the benefits of using that cooperative learning in classrooms can also be observed with other variables. The large body of empirical evidence regarding the contribution of cooperative methods for achievement (Hattie, 2008; Slavin, 2014), self-esteem (Johnson & Johnson, 1989), motivation (Johnson, et al., 2014), and peer relationships (Roseth, Johnson, & Johnson, 2008) means that real value exists in supporting teachers in the implementation of these methods in their daily teaching. Nevertheless, it might be not sufficient to propose that pupils/students merely cooperate; rather, the way the teacher structures social interactions in groups is important to favor all students' learning. Our study proposes a pedagogical cooperative learning method that can be used in classrooms to improve the organization of social interactions in heterogeneous work groups in order to support understanding and learning from all students participate in groups.

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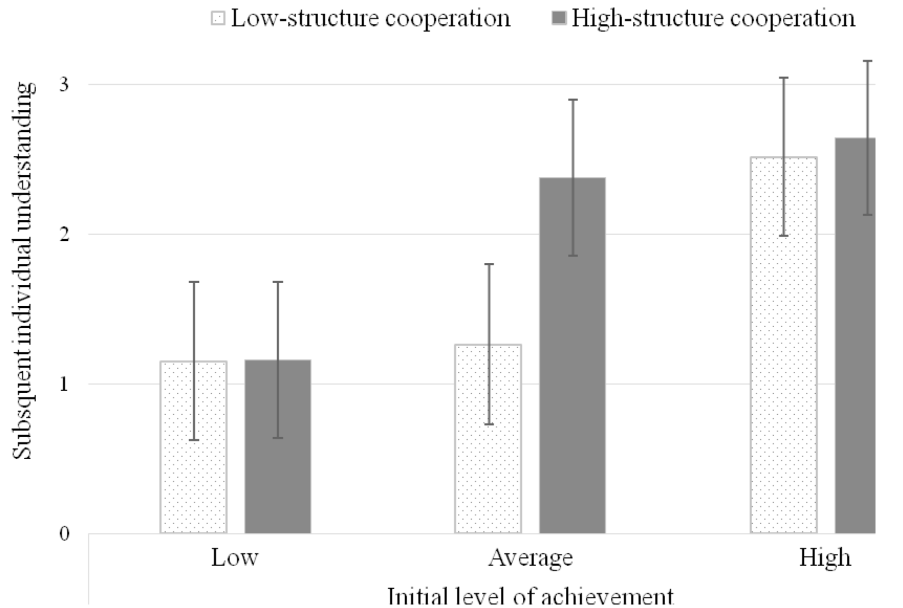


Figure 1. Individual level of understanding as a function of initial level of achievement and structure of cooperation. First set of analyses.

Notes: Error bars represent 95% confidence intervals of the estimated means.

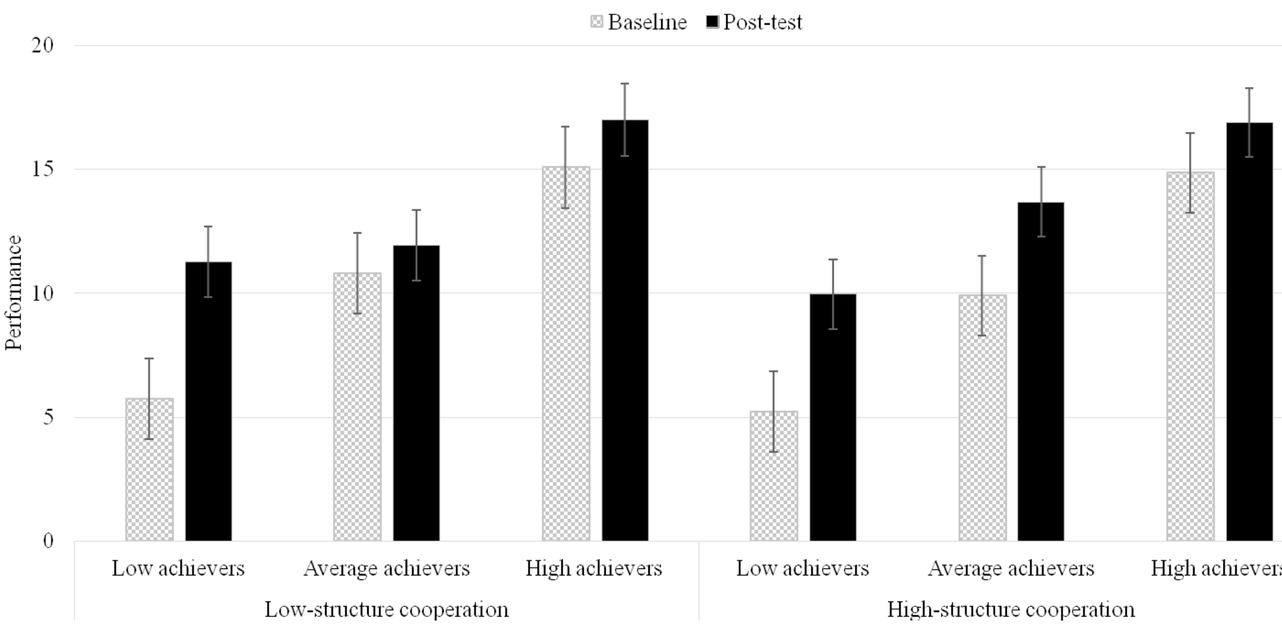


Figure 2. Baseline-to-post test progress as a function of initial level of achievement and structure of cooperation. First set of analyses.

Notes: Error bars represent 95% confidence intervals of the estimated means.

Appendix A. Card rule 1: "Responsible of reasoning".

Responsible of reasoning

EXPLAINING THE THREE REASONING

"I want to share equitably 4 identical pizzas in 3 guests"

Reasoning 1	**Reasoning 2**	**Reasoning 3**
We can give everyone a pizza and then cut the last one and give a part to each of them.	We can cut each pizza into three parts and distribute a portion of each pizza to every guest.	We can cut all the pizza in three parts and give three parts, representing a whole pizza and a part of another pizza to each guest.
 $1+\frac{1}{3}$	 $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$	 $\frac{4}{3}$
Addition of whole number and fractions	Addition of fractions	Fractional writing

- I explain my reasoning for writing.
- I make sure that everyone understands :
  - "Did you have some questions?"
  - "Is that is enough clear to you?"

Appendix B. Card Rule 2: "Communicate with appropriate vocabulary".

Responsible of vocabulary

COMMUNICATE WITH APPROPRIATE VOCABULARY

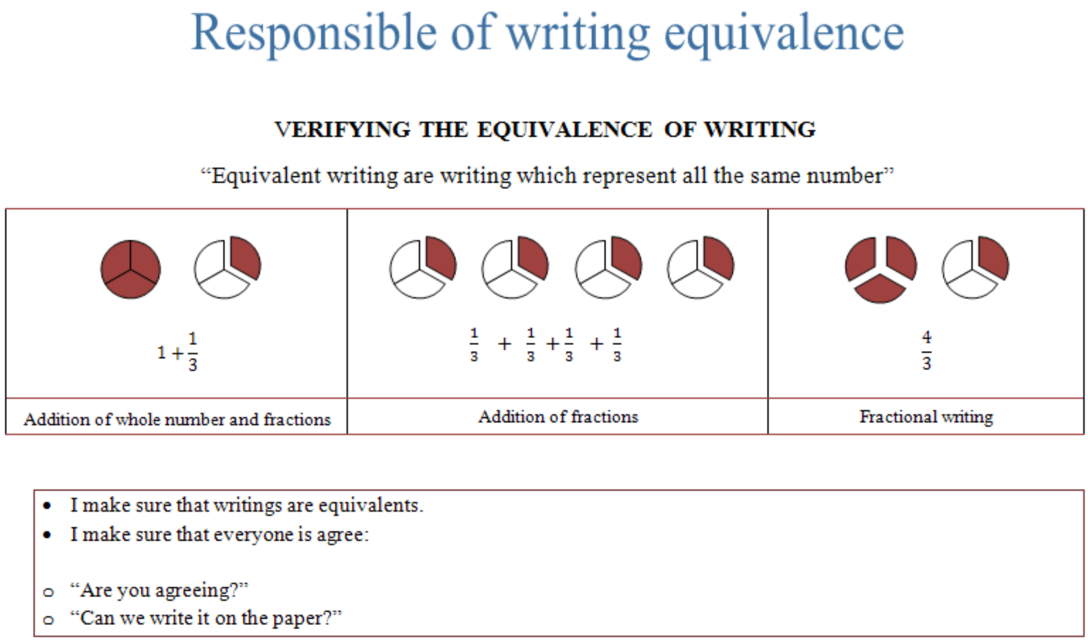
$$\frac{1}{3}$$

~~One on three~~

⇨ **A third**

- I write the common response on the paper.
- I make sure that my friend uses the appropriate vocabulary.

Appendix C. Card rule 3: "Responsible of writing equivalence".



Acknowledgments:

This work was supported by the Swiss National Science Foundation under Grant CR-SII1_14172 and the French University Institute and the Région Auvergne.

We wish to thank Jean-Luc Dorier for his help in designing the mathematical task.

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Структурално кооперативно учење као средство унапређења просечних постигнућа ученика приликом учења разломака из математике

У основној школи учење разломака је најважнија област у настави математике. Усавршавање основних процедура које се тичу разломака представља тешкоћу за многе ученике. Циљ овог истраживања је да се представи структурално кооперативно учење као средство које може да унапреди учење ученика, а ово се посебно односи на просечне ђаке. У претходном истраживању утврђено је да хетерогене групе (у којима су ученици који постижу мали, просечан и велики успех) могу да буду штетне за ученике који имају просечна постигнућа, јер су они искључени из односа наставника и ученика који имају лоша или добра постигнућа. Ово истраживање предлаже да се структурише интеракција ради побољшања постигнућа просечних ученика у хетерогеним групама.

Приликом овог истраживања, сто осам ученика петог разреда радило је заједно у хетерогеним тријадама које су сачињене према резултатима на иницијалном тесту (један ученик са ниским нивоом постигнућа, један са средњим и један са високим). Тријаде су насумично биле изложене нискоструктурисаним и високоструктурисаним условима кооперативног учења. У свим тријадама ученицима је било наложено да раде заједно, водећи рачуна о свом учењу и учењу својих партнера.

Математички задатак је укључио три вештине: 1) разумевање разломачког резоновања (сабирање целог броја и разломка, сабирање разломака, писање разломака); 2) схватање еквиваленције писања разлике; и 3) способност коришћења адекватног вокабулара. Да би се радило на поимању разломака, предложили смо типичне вежбе које се користе у националном курикулуму. Тријаде су морале да изразе дужину једног сегмента, користећи три лењира са различитим подјединицама и поштујући три математичке вештине које се односе на разломке. Стандардно мерење је било загарантовано различитим подјединицама под именом „три лењира“. Ученици су морали да напишу дужину сегмента користећи што је могуће више израза, уз адекватан речник. Такође, морали су и да провере да ли су сви изрази били еквивалентни. Од њих се захтевало да користе лењире да би измерили сегмент. Три социјалне одговорности су такође обухваћене: проверавање да ли су сви разумели, потврђивање да се сви слажу око заједничког одговора и обавештавање о заједничком одговору. Ученици су извештавали о заједничком одговору на групном листу (позитиван циљ и независност). Били су замољени да

подстакну једни друге и да објасне резоновање (конструктивна интеракција). Такође, било им је речено да ће радити индивидуални тест после рада у групи (индивидуална одговорност). Ова коопертивна упутства су дата у оба случаја.

У условима ниске структурисаности материјал је подељен свим ученицима (то јест сваки ученик је имао три различита лењира). Ученици су морали да примене три математичке вештине (објашњавање резоновања, проверавање еквивалентности израза и коришћење адекватног речника). Морали су да продискутују о одговорима у тријадама користећи све вештине и лењире; морали су да буду сигурни да су сви разумели и да онда саопште заједничке одговоре. Организовали су рад у групи како год су желели.

Полази се од премисе да ученици који имају просечна постигнућа могу да буду мање активни у хетерогеној групи и да преузимање активне улоге приликом објашњавања представља главни елемент у математици и веома велики структурално-кооперативни услов за учење који има за циљ да сви ученици у тиму буду укључени у математичке дискусије и групне одлуке. Уз то, увели смо дистрибуцију материјала, комплементарну експертизу и мењање одговорних ученика током вежбе. У условима високе структурисаности, материјали су били подељени међу ученицима у свакој тријади (то јест један лењир по особи) и свако би постао експерт за тај лењир пре него што објасни стечене вештине вршњацима у одређеним тријадама. Посебно смо направили листу одговорности које су се базирале на циљним математичким вештинама и циљним социјалним одговорностима и предложили им да ученици наизменично врше дужности током вежбе. Ова процедура је предложена да би се ојачала индивидуална одговорност и позитивна међузависност.

После вежбе у тријадама процењивано је индивидуално разумевање и онда су ученици расправљали о индивидуалним завршним задацима са разломцима. Опсервацијом суобухваћени ученици (то јест ниво 1) који су били у тријадама (то јест ниво 2) и они који су били у учионицама (то јест ниво 3). Резултати су показали да високо структурисано кооперативно учење даје примат разумевању задатог задатка, нарочито за ученике просечних способности. Штавише, ученици на свим нивоима су напредовали од иницијалног теста до завршног теста. Заправо, ученици са малим и великим постигнућима су подједнако напредовали код оба услова, док су просечни напредовали више код високо структурисаних услова.

Када се узму заједно, ови резултати потврђују да више структурисани (у односу на мање) бивају ефектнији за просечне ученике него за оне који постижу горе или боље резултате од просечних, и који могу да имају користи од сарадње без обзира на структурни ниво. Још једна важна чињеница добијена овом студијом је да ниво структуре нема ефекта на разумевање или на напредовање ученика са малим и великим постигнућима. Ови налази говоре да структура која подразумева да сви ученици буду социјално и когнитивно укључени током групног рада представља круцијалну компоненту која омогућава ученицима просечних постигнућа да имају користи од сарадње. Наша студија предлаже педагошки кооперативни метод учења који може да се користи у учионици да би се побољшала организација социјалне интеракције у хетерогеним групама и да би се подржало разумевање и учење свих ученика који учествују у групама.

Кључне речи: кооперативно учење, структура, учење разломака, ученици просечног постигнућа, математика.